Chapter layout

❶ KEY CONCEPTS - Each chapter begins with a list of the key ideas covered in each section of the chapter.

❷ EXPLORATION - Commonly, the *Key Concepts* of a chapter are first introduced in an *Exploration*, where students work in or out of class, individually or in groups. **These can form the basis of rich assessment tasks.**

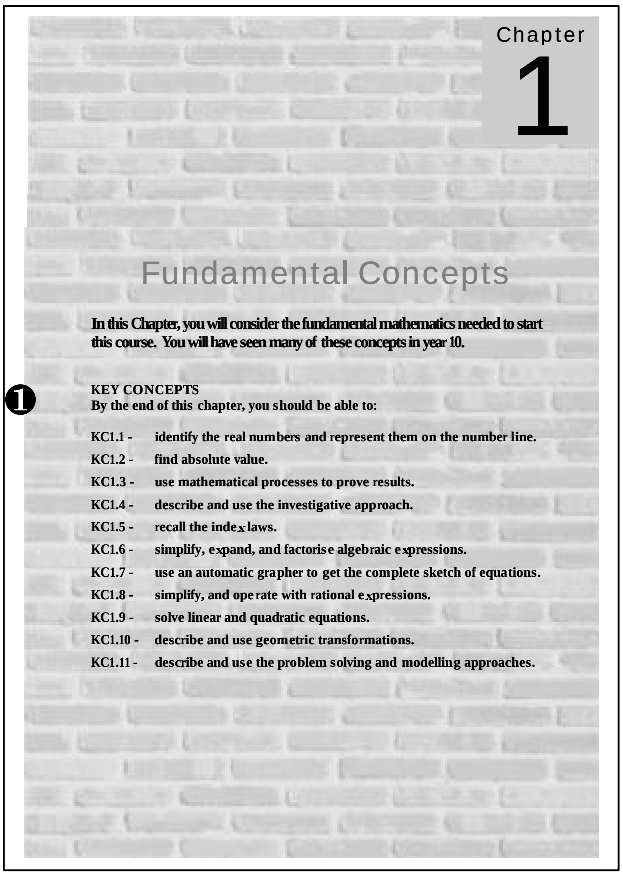
❸ CONCLUSION - After each *Exploration*, there is a discussion about the mathematics covered in the *Exploration*.

❹ CHECK YOUR UNDERSTANDING - After each *Key Concept* is explored and discussed, students are given an opportunity to check how well they have learnt the idea in the Check your understanding. There is one of these for each *Key Concept*.

❺ ANSWERS - To assist with the student's understanding, the answers immediately follow each of the Check your understanding's.

❻ CHALLENGE FOR CHAMPIONS - These are for students who feel that they have mastered the *Key Concepts* of the section and are looking for more challenging tasks. **These can also form the basis of rich assessment tasks.**

❼ ENRICHMENT - Throughout the textbook, there are references to *Enrichment* tasks and links at **MIC**, the web site that accompanies the text. **These can also form the basis of rich assessment tasks.**

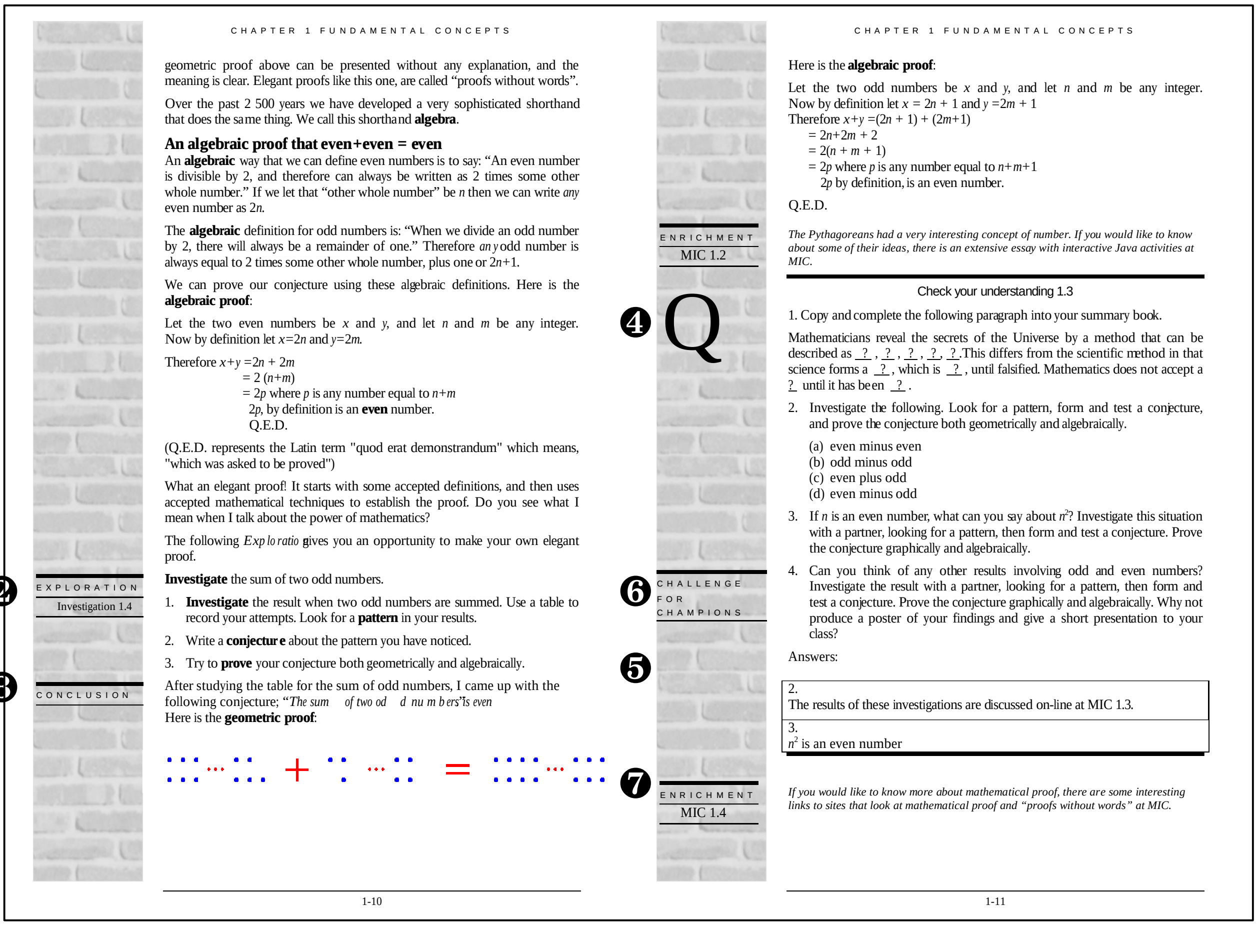

Chapter 1

Fundamental Concepts

In this Chapter, you will consider the fundamental mathematics needed to start this course. You will have seen many of these concepts in year 10.

❶ **KEY CONCEPTS**
By the end of this chapter, you should be able to:

KC1.1 - identify the real numbers and represent them on the number line.
KC1.2 - find absolute value.
KC1.3 - use mathematical processes to prove results.
KC1.4 - describe and use the investigative approach.
KC1.5 - recall the index laws.
KC1.6 - simplify, expand, and factorise algebraic expressions.
KC1.7 - use an automatic grapher to get the complete sketch of equations.
KC1.8 - simplify, and operate with rational expressions.
KC1.9 - solve linear and quadratic equations.
KC1.10 - describe and use geometric transformations.
KC1.11 - describe and use the problem solving and modelling approaches.


CHAPTER 1 FUNDAMENTAL CONCEPTS

geometric proof above can be presented without any explanation, and the meaning is clear. Elegant proofs like this one, are called "proofs without words".

Over the past 2 500 years we have developed a very sophisticated shorthand that does the same thing. We call this shorthand **algebra**.

An algebraic proof that even+even = even

An **algebraic** way that we can define even numbers is to say: "An even number is divisible by 2, and therefore can always be written as 2 times some other whole number." If we let that "other whole number" be n then we can write *any* even number as $2n$.

The **algebraic** definition for odd numbers is: "When we divide an odd number by 2, there will always be a remainder of one." Therefore *any* odd number is always equal to 2 times some other whole number, plus one or $2n+1$.

We can prove our conjecture using these algebraic definitions. Here is the **algebraic proof**:

Let the two even numbers be x and y, and let n and m be any integer. Now by definition let $x=2n$ and $y=2m$.

Therefore $x+y = 2n + 2m$
$= 2(n+m)$
$= 2p$ where p is any number equal to $n+m$
$2p$, by definition is an **even** number.
Q.E.D.

(Q.E.D. represents the Latin term "quod erat demonstrandum" which means, "which was asked to be proved")

What an elegant proof! It starts with some accepted definitions, and then uses accepted mathematical techniques to establish the proof. Do you see what I mean when I talk about the power of mathematics?

The following *Exploration* gives you an opportunity to make your own elegant proof.

❷ EXPLORATION
Investigation 1.4

Investigate the sum of two odd numbers.

1. **Investigate** the result when two odd numbers are summed. Use a table to record your attempts. Look for a **pattern** in your results.
2. Write a **conjecture** about the pattern you have noticed.
3. Try to **prove** your conjecture both geometrically and algebraically.

❸ CONCLUSION

After studying the table for the sum of odd numbers, I came up with the following conjecture; "*The sum of two odd numbers is even*"
Here is the **geometric proof**:

1-10

CHAPTER 1 FUNDAMENTAL CONCEPTS

Here is the **algebraic proof**:

Let the two odd numbers be x and y, and let n and m be any integer. Now by definition let $x = 2n + 1$ and $y = 2m + 1$
Therefore $x+y = (2n + 1) + (2m+1)$
$= 2n+2m + 2$
$= 2(n + m + 1)$
$= 2p$ where p is any number equal to $n+m+1$
$2p$ by definition, is an even number.

Q.E.D.

ENRICHMENT
MIC 1.2

The Pythagoreans had a very interesting concept of number. If you would like to know about some of their ideas, there is an extensive essay with interactive Java activities at MIC.

❹ Q

Check your understanding 1.3

1. Copy and complete the following paragraph into your summary book.

Mathematicians reveal the secrets of the Universe by a method that can be described as _?_, _?_, _?_, _?_, _?_. This differs from the scientific method in that science forms a _?_, which is _?_, until falsified. Mathematics does not accept a _?_ until it has been _?_.

2. Investigate the following. Look for a pattern, form and test a conjecture, and prove the conjecture both geometrically and algebraically.
   (a) even minus even
   (b) odd minus odd
   (c) even plus odd
   (d) even minus odd
3. If n is an even number, what can you say about n^2? Investigate this situation with a partner, looking for a pattern, then form and test a conjecture. Prove the conjecture graphically and algebraically.
4. Can you think of any other results involving odd and even numbers? Investigate the result with a partner, looking for a pattern, then form and test a conjecture. Prove the conjecture graphically and algebraically. Why not produce a poster of your findings and give a short presentation to your class?

❻ CHALLENGE FOR CHAMPIONS

Answers:

❺

2.
The results of these investigations are discussed on-line at MIC 1.3.

3.
n^2 is an even number

❼ ENRICHMENT
MIC 1.4

If you would like to know more about mathematical proof, there are some interesting links to sites that look at mathematical proof and "proofs without words" at MIC.

1-11

❽ PROJECT - Each chapter ends with a suggested *Project* that encourages the student to synthesise the *Key Concepts* of the chapter. It is suggested that the *Project* be completed in the **Technical report** genre. There are suggested ideas for the projects and a link to MIC. **These can also form the basis of rich assessment tasks.**

❾ CHAPTER REVIEW - In addition to the *Project*, each chapter ends with a battery of graded questions that test the student's mastery of the *Key Concepts.* The review ranges from simple questions that check the student's **Knowledge and Understanding**, and **Skills and Procedures**, through to complex questions that give them an opportunity to **Utilise** their knowledge and do **Explorations and Problem Solving**.